Optimization of the Dynamic Vehicle Routing Problem with Increasing Scale

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Abstract—The developing retail industry in China presents colony type characteristics, i.e., the amount of stores is increasing, the order per store is small but the total demand is large. This paper examines the dynamic vehicle routing problem with increasing scale (DIS-VRP) in this setting. We present a new model for this problem, which incorporates large and increasing scale and uncertainty. A three-phase heuristic algorithm is proposed to solve the model. The application of our model and algorithm in Meiyijia shows that it is effective and efficient.

Keywords—vehicle routing problem; route adjustment rate; scale increasing; retail

I. INTRODUCTION

With the rapid economic development in China, the retail industry has developed tremendously in the past decade, and presents colony type characteristics, i.e., the number of stores is increasing, the orders per store is small but the total demand is large. To gain the benefit of the economies of scale, usually a unified distribution center serves all the stores in a region. With the transportation cost taking up to 50 percentage of the whole distribution cost, it is important to reduce the transportation cost which makes the vehicle routing problem crucial in the real business practice [1]. This paper studies the new problem which we called dynamic vehicle routing problem with an increasing scale (DIS-VRP), i.e., the number of stores is more than one thousand and keeps increasing stochastically.

Increase in the number of stores makes the route change every day. However, the driver familiarity with routes and customers conduces to less error, exceptional cost and more customer satisfaction. It’s beneficial to control route adjustment by creating regular or consistent routes for the DIS-VRP that assign the same driver to the set of customers to serve them through the same routes. Such consistent routes are easy to adapt to the realization of the daily uncertainty and help logistics improve the service quality.

We also focus on the detailed analysis of transportation cost, including not only vehicle fee, but also labor cost. From practical experience, on one hand, labor cost is an important part of delivery cost, while it hasn’t been contained by the literatures on vehicle routing problem. On the other hand, various vehicle costs are considered, including fuel, supplies (e.g. tyre) expense, insurance, and maintenance cost etc.

A. Literature Review

The vehicle routing problem (VRP) variants relevant to our work are VRP with stochastic customers (VRPSC) and with multi-objective (MVRP).

Dynamic vehicle routing problem with increasing scale (DIS-VRP) is related to the VRPSC. A number of models and algorithms for VRPSC allow recourse actions to adjust a priori solution after the uncertainty is revealed. Different recourse actions have been proposed in the literature, such as remove customers who do not require service [2], complete reschedule for occurring customers [3], or return to the warehouse when the capacity is exceeded [4]. Recent work by Hvattum, Lokketangen and Laporte uses a heuristic method to solve a dynamic and stochastic VRP problem, where master scenarios are generated, solved heuristically, and combined to form an overall solution [5]. Recently, Sungur, Ren, Ordóñez, Dessouky and Zhong develop scenario-based stochastic programming to generate a master plan and daily schedules considering the similarity of routes in each scenario [6].

Other strand of literature related to DIS-VRP is about the MVRP, which is often studied in school bus routing [7] [8], waste collection [9], and hazardous products transportation [10] [11]. The three main solution strategies for multi-objective problems are the Pareto approach, which identifies a set of non-dominated solutions, the scalar method which uses mathematical transformation to minimize a weighted linear aggregation of the objectives, and other technique that considers the different objectives separately. We refer to Jozefowiez, Semet and Talbi [12] for a recent survey of these approaches to the MVRP.

B. Organization of the Paper

In this paper, we formulate the problem as a mixed integer linear program by the scalar technique. We then develop a three-phase heuristic based on district group for its template solution and daily schedules, and computational results show that our approach can yield high quality solutions within reasonable running times. The rest of paper is organized as follows. The mathematical model is described in Section II. The three-phase heuristic is described in Section III, followed by computational results in Section IV and by conclusions in Section V.
II. Mathematical Problem Description

We first give a more detailed description of the dynamic vehicle routing problem with increasing scale, and then formulate it as a mixed integer program to capture the problem more precisely.

A. Problem Description and Analysis

The dynamic vehicle routing problem with increasing scale (DIS-VRP) is a capacitated vehicle routing problem having more than one thousand customers. Considering all the requests, available vehicles and human resources, logistics center plans and constructs daily schedules to meet the delivery requests at minimum cost.

Distribution network DIS-VRP is motivated by the operations of a third-party logistics that serves Meiyijia, which is probably the largest company in convenient chain industry. Fig. 1 shows the locations of known customers and the distribution center. Most stores are located in dense urban area or business district, while the warehouse is in the remote area as a result of obtaining lower warehouse cost. Therefore, those stores nearby the distribution center are in dense and those far away from warehouse locate in loose.

![Figure 1. Locations of stores and a warehouse in Meiyijia.](image)

Cost From real practice, the transportation cost mainly contains labor cost and vehicle fee during the delivery job. One driver and one carrier are assigned to each vehicle in short routes, whereas in some remote routes, there are two drivers and one carrier. Vehicle cost consists of static part, which is only related to the number of vehicles, and dynamic part which is positive correlation with distance. Static cost mainly contains the vehicle depreciation, maintenance, insurance and various taxes, while the dynamic part mainly contains fuel, road toll, supplies (e.g. tyre) expense and accidents.

It is beneficial to analyze and denote the detailed labor cost. As to static and dynamic cost of vehicle, parameter which is the weighted sum of all the above mentioned is given respectively.

Dynamic increasing scale New stores opened, old stores closed and stores position changed results in routes adjustment. To demonstrate the routes change in the above three conditions, Fig. 2 furnishes a small DIS-VRP example consisting of one depot represented by a square, seven stores represented by a round, and three routes represented by a directed graph. One should be noticed is that we measure routes change with respect to master routes, which is depicted in (a); when a new store opened, it’s inserted into the right route and make routes change as (b); when a store position changed, routes change as (c); when a store closed, it’s removed from routes and routes change as (d).

![Figure 2. A small instance of the dynamic vehicle routing problem with increasing scale. (a) Initial solution, (b) Solution for a new store opened, (c) Solution for a store position changed, (d) Solution for a store closed.](image)

B. Mathematical Formulation

The daily schedule of DIS-VRP is a special case of the CVRP. Denote the number of available vehicles by $K$, each maximum capacity of vehicle is $Q_k$. The set of all locations is $N = \{0, 1, 2, \ldots, n\}$, and the warehouse is located at 0. We use $T$ to represent period, $t = 0$ to present the beginning of period, and the set of all locations changes to $N_t$. The set of all available workers is $E$, and the parameter $e_i$ represents the number of workers in vehicle $k$. Let $q_{ij}$ be the demand at customer $i$, and $d_{ij}$ be the distance between location $i$ and $j$. Denote the maximum number of stores a vehicle served at one time by $\Omega$, including the departed and returned warehouse.

Define variable:

\[
x_{ik} = \begin{cases} 
1, & \text{vehicle } k \text{ travels from location } i \text{ to } j \\
0, & \text{others}
\end{cases}
\]

The constraints are defined as follows:

\[
\max_{i \in N_t, j \in N_t} (q_{ij}) < \min_{k \in K} (Q_k) \quad (1)
\]

\[
\sum_{i \in N_t} \sum_{j \in N_t} x_{ik} \leq Q_k \quad \forall k \in K \quad (2)
\]
\[
\sum_{j \in N_k}\sum_{i \in N_j} x_{ijk} = \sum_{j \in N_k} x_{i0k} \leq 1 \quad i = 0, \forall k \in K \tag{3}
\]
\[
\sum_{i \in N_k} x_{ijk} = 1 \quad \forall j \in N_0 \tag{4}
\]
\[
\sum_{j \in N_k} x_{ijk} = 1 \quad \forall i \in N_0 \tag{5}
\]
\[
0 \leq \sum_{i \in N_k} \sum_{j \in N_j} x_{ijk} \leq \Omega \quad \forall k \in K \tag{6}
\]
\[
\sum_{x \in E} c_i \leq |E| \tag{7}
\]

Constraints (1) ensure that each customer’s demand is not exceed the vehicle capacity. Constraints (2) guarantee the vehicle capacity limit is not exceeded. Both (1) and (2) are the vehicle capacity limit. Constraints (3) state that each vehicle should start and end its route at the warehouse. Constraints (4) and (5) ensure that each customer must be served only once, by only one vehicle at one distribution job. The maximum number of stores of route is guaranteed by (6). Constraints (7) ensure workers participating in delivery job must not exceed the available numbers.

The first objective, minimizing the total transportation cost on day \( t \in T \), can be formulated as
\[
f'_t = \sum_{l \in K} c_{kl} \cdot \sum_{i \in N_l} \sum_{j \in N_j} x_{ijk} + \sum_{i \in N_l} c_i + \sum_{i \in N_l} \sum_{j \in N_j} \sum_{k \in K} c_{ak} d_{jk} x_{ijk} \tag{8}
\]

The first part of (8) is human cost, including basic wage, push money in correspondence to workload, and district allowance. The parameter \( x_i \) is related to the capacity and distribute region. Let \( c_{af} \) be basic wage, parameter \( \alpha \) be commission coefficient and \( \varepsilon \) be region allowance. Workers salary can be formulated by
\[
c_c = c_{af} + \alpha \cdot \sum_{i \in N_l} \sum_{j \in N_j} x_{ijk} + \varepsilon \tag{9}
\]

The second part of (8) is static vehicle cost, denoted by \( c_{vk} \). The last part of (8) is dynamic vehicle cost, multiplied by unit cost and distances. Let \( c_{vk} \) be unit cost, the cost of vehicle \( k \) travels one kilometer.

The second objective, minimizing the routes changed. Based on the initial solution, let the route adjustment ratio be the number of changed routes divided by all the routes in initial solution over a period of time. Number the routes by \( L_t \). Denote vehicle \( k \) run the route \( l \) at day \( t \) by the vector \( R_{lk}^t \), component of which is location. Let function \( \phi(X) \) be the nonzero numbers of vector \( X \). The route adjustment ratio can be formulated as:
\[
f'_t = \frac{\sum R_{lk}^t - \sum R_{lk}^0}{\sum R_{lk}^t - \sum R_{lk}^0 + \phi(\sum R_{lk}^0)} \tag{10}
\]

As mentioned, scalar techniques and the Pareto method are the two main solution strategies for multi-objective optimization. However, the Pareto method is not appropriate in the dynamic context. Because even if it were possible to find a set of Pareto optimal solutions, it would be necessary to implement at least one of these before the next period planning, without guidelines on how to make the selection. Therefore, we have opted to implement the scalar method with weights, 1 and \( w_1 \), for objective \( f'_1 \) and \( f'_2 \), respectively, and we give the aggregate objective
\[
f' = f'_1 + w_2 f'_2 \tag{11}
\]

### III. A THREE-PHASE HEURISTIC

We propose a three-phase heuristic (TPH) based on district partitioned to handle the large scale aspect of the problem. Phase I groups all customers by districts and arterial streets. Denote a user-defined parameter to adjust and balance the workload among the groupings. In phase II, master routes are constructed by means of a sweep heuristic [13], and are optimized by a variable neighborhood search heuristic. At the beginning of each period, phase I and phase II are implemented to solve the problem as a large scale deterministic vehicle routing problem. In phase III, postoptimize the master routes to formulate a least-cost feasible route solution which is performed on day \( t \). This three-phase scheme is summarized in Algorithm 1.

\begin{itemize}
\item \textbf{Algorithm 1. TPH framework}
\item 1: Input: the set of known customers \( N_0 \) at the beginning of period and the set of customers \( N_t \) within period \( T_t \)
\item 2: Output: the routing plan \( R = \{ R^1, R^2, \ldots, R^n \} \) for horizon \( T_t \)
\item 3: \{ \( G_1^t, \ldots, G_p^t \) \} \leftarrow \text{GroupCustomers}(N_0) \text{//Phase I}
\item 4: for \( m = 1 \) to \( p \) do
\item 5: \( S^t \leftarrow \text{RouteCustomer}(G_m^t) \text{//Phase II}
\item 6: end for
\item 7: for \( t = 1 \) to \( |T_t| \) do
\item 8: \( S^t \leftarrow \text{Optimize}(S^t, N_t) \text{//Phase III}
\item 9: end for
\end{itemize}

\begin{itemize}
\item \textbf{A. Phase I: Customer Grouping}
\item The customer grouping phase attempts to determine a good set of groups on both the known customers and the uncertainty in customers changed over a period of time. Based on the administrative district and high street, perform the division to reduce the complexity of large scale problem and make sure that customers from different city in different groups. Moreover, to balance the workload on different groups, let constant \( \delta \) be the capacity of groups defined at the beginning of period. Whenever the number of customers in a group is more than \( \delta \), split up the group into small groups. This procedure is described as Algorithm 2.
\item \textbf{Algorithm 2. Phase I (Customer grouping)}
\end{itemize}
Phase I (Variable neighborhood search)

The aim of the Phase II is to construct master routes and the objective is to minimize the total travel cost. In this phase, the problem is solved by the variable neighborhood search heuristic (Algorithm 3) made up of two components: initialization and local search. Initial solution is first constructed by a sweep heuristic. The local search phase is based on a tabu search (TS) algorithm that uses simple relocation moves to relocate the customer in the same route and exchanges to transfer customers from their route to another route. For each customer, all possible relocation and exchange positions are attempted and the one leading to the minimum value of (11) is selected. One should be mentioned is that postoptimization only acts on those groups changed so that it costs less time to formulate a set of least-cost routes for daily schedule.

**Algorithm 3. Phase II (Variable neighborhood search)**

1: Input: the set of grouped customers \( G^0 \)
2: Output: the solution \( S^*_u \)
3: \( S \leftarrow \text{SweepHeuristic}(G^0_u) \)
4: \( S^*_u \leftarrow S \)
5: \( \text{iteration} \leftarrow 0 \)
6: while \( \text{iteration} < \alpha \) do
7: \( (S, G^0_u) \leftarrow \text{TabuSearch}(S, G^0_u) \)
8: \( \text{iteration} \leftarrow \text{iteration} + 1 \)
9: end while
10: return \( S^*_u \)

**C. Phase III: Postoptimization**

Phase III aims to minimize the total cost on day \( t \). First of this phase, group the different customers from day \( t \) into the groups given by the result of Phase I. Secondly, for each customer changed in groups adjustment, all possible insertion and exchange positions are attempted and the one leading to the minimum value of (11) is selected. One should be mentioned is that postoptimization only acts on those groups changed so that it costs less time to formulate a set of least-cost routes for daily schedule.

**IV. EXPERIMENTAL RESULTS**

In Meiyijia case, the number of stores has increased from 2380 to more than 3000 in the past year and almost 53 new stores opened per month. Fig. 3 demonstrates the statistical number of stores located in different city, while the detailed distribution of the customers and warehouse was illustrated in Fig. 1. Obviously the warehouse locates at Dongguang (denoted by DG) having the largest number of stores.

**Figure 3. Distribution of the number of stores locating in different city**

Three types of vehicles were considered in our test, with 1.5 tons, 3 tons and 5 tons in terms of weight capacity, respectively. However, volume is used much more than weight with respect to vehicle capacity in real practice. Hence we were opted to maximum volume, which is represented in row 2 of Table I, to present the vehicle capacity. More detailed parameters and data used relevant to vehicle were shown in Table I. The available vehicles and workers were given in row 3 and row 6, respectively. Row 4 and 5 are the static fee and unit dynamic cost, respectively.

**TABLE I. DETAIL VEHICLE DATA AND PARAMETERS**

<table>
<thead>
<tr>
<th>Items</th>
<th>Load Capacity(tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Volume capacity ( Q_k )</td>
<td>15.7</td>
</tr>
<tr>
<td>Number of vehicles (</td>
<td>K</td>
</tr>
<tr>
<td>Static fee ( c_{fa} )</td>
<td>42.8</td>
</tr>
<tr>
<td>Dynamic fee ( c_{du} )</td>
<td>0.50975</td>
</tr>
<tr>
<td>Number of workers ( e_i )</td>
<td>2</td>
</tr>
</tbody>
</table>

Basic wage \( c_{fa} \) and commission coefficient \( e_i \) were set to 60 and 2.2, respectively. We use the real travel distances instead of Euclidian distances, and according to the district
distances from warehouse, we set district allowance ε to 0, 45 and 65, respectively. The average demand of the orders is 1.53, and the maximum number of stores a route is 30.

Parameters δ in Phase I and ω in Phase II of the TPH were set to 600 and 100 respectively. To control the route adjustment ratio effectively, it is necessary to take into account the possibilities of stores changed over a period of time when executing the Phase I and Phase II to make the flexible template routes. The estimated for stores changed is obtained from both the increment in the previous period and the expansion of company, and is updated adaptively for each period.

The model and three-phase heuristic just described was developed into Transportation Management System (HDTMS) of Heading Ltd, which is already used for distribution center to formulate feasible and flexible routes in the daily schedules. However, considering the commercial confidential, we have limited permission to present the computational results from real practice and only a few results are showed in this paper. Fig. 4 presents a daily scheduling screenshot from HDTMS, and Fig. 5 illustrates the result routes at one time.

![Figure 4. Scheduling in Transportation Management System](image)

![Figure 5. Result routes in Transportation Management System](image)

Meiyijia have many achievements in logistics delivery with the help of HDTMS in the past year, such as the time on planning daily schedules reduced from 2 hours to 40 minutes and on daily job decreased from 18 hours to 9 hours, the number of workers reduced nearly by 50% and of vehicles decreased by 17%, and the total transportation cost decreased almost by 22% and so on. It is obviously that our approach provides high quality solutions.

**V. CONCLUSION**

We have considered a real-life dynamic increasing scale and multi-objective routing problem encountered by a large distributor operating in China. The increasing is uncertain so that route adjustment ratio is assigned to balance the route change over a period of time, and the problem considers two objectives, including minimization of the total travel cost, which consists of labor cost and various vehicle fees. We have proposed a mixed integer programming formulation for the problem, and we have developed a three-phase heuristic algorithm to solve the problem. The main idea of the heuristic is to group customers to decrease the complexity of problem, and to route customers by local search so that the overall travel cost can be minimized efficiently. The multiple objectives are handled by the scalar technique. The scheme was implemented on real-life data. Application results present that the proposed heuristic provides very high quality solutions within a reasonable running time.

**REFERENCES**